THE UNITS METHOD

Most of us at one time or another have had to juggle a formula or two or convert one unit of measurement into another. If you're like me, sometimes it is difficult to remember which number gets multiplied by which… or, were we supposed to divide those numbers in the first place?

Thank heavens for reference books (and the Internet!) but sometimes the formula or conversion factor we need is just not readily available. There is a little trick to help keep computations straight. I call it the "Units Method." By keeping the units that something is measured in as part of the equation, and by processing them along with the numbers, many calculations become intuitive. (This method is also known as Unit Analysis, Dimensional Analysis, Factor-Label Method or Unit-Factor Method; more about this later.)

A couple of examples will show how the Units Method works:

Example #1

Velocity is measured in units of distance over units of time, e.g., miles-per-hour, feet-per-second, etc. Written as "miles/hour" or "feet/second", the "per" is given the division symbol. The unit of distance is in the numerator (top), and the unit of time is in the denominator (bottom.)

$$\frac{miles}{hour} \qquad \frac{feet}{\text{second}}$$

Let's say we wanted to compute how far we would travel if we drove at 55 miles-per-hour for two hours. We would use the simple formula:

(1.1) $$D = (V) \times (t)$$

where (D) is the distance traveled in miles, (V) is the velocity in miles-per-hour, and (t) is the time in hours. Inserting numbers and units into equation (1.1) we get:

(1.2) $$D = \left(\frac{55 miles}{hour}\right) \times \left(\frac{2 hours}{1}\right)$$

Notice that the unit "hours" cancels out:

(1.3) $$D = \left(\frac{55 miles}{\cancel{hour}}\right) \times \left(\frac{2 \cancel{hours}}{1}\right)$$

Multiplying through, we are left with "miles," (i.e., distance)

(1.4) $$D = \left(\frac{55 miles}{1}\right) \times \left(\frac{2}{1}\right) = 110 miles$$

Of course, simple examples like this can be done in the head, but the practice of putting them onto paper helps when the formula isn't so intuitive.

A couple of rules to keep in mind:

- Rule A: Singular and plural forms of a unit, as in the "hour" "hours" example above, are generally interchangeable.
- Rule B: Hertz (Hz.) for Units calculations is generally better stated by it's equivalent: "cycles/second."
- Rule C: Multiplying by the reciprocal of a number is the same thing as division
- Rule D: Conversion factors and their reciprocals can be strung together and used at will to get the desired units.

Example #1 above illustrated the Rule A. The following examples will illustrate the other three.

Example #2:

The relationship between velocity (V), wavelength (λ), and frequency (F) is:

(1.5) $$V = (\lambda) \times (F)$$

Where velocity (V) is in meters/second, wavelength (λ) is in meters/cycle, and frequency (F) is in cycles/second (Rule B, instead of Hz). Plugging units into eqn. (1.5) gives:

(1.6) $$V = \left(\frac{meters}{cycle}\right) \times \left(\frac{cycles}{\text{second}}\right)$$

Notice that "cycles" on the right hand side of the equation cancel out and we are left with meters/second which is the unit of velocity:

(1.7) $$V = \left(\frac{meters}{\cancel{cycle}}\right) \times \left(\frac{\cancel{cycles}}{\text{second}}\right)$$

(1.8) $$V = \left(\frac{meters}{\text{second}}\right)$$

If we wanted to solve for wavelength we would juggle the velocity formula (1.5) for λ:

(1.9) $$\lambda = \left(\frac{V}{F}\right)$$

or,

(1.10) $$\lambda = \text{V} \div \text{F}$$

Inserting units we have:

(1.11) $$\lambda = \left(\frac{meters}{\text{sec}}\right) \div \left(\frac{cycles}{\text{sec}}\right)$$

Since we can multiply by the reciprocal of (F) instead of dividing (Rule C), we have:

(1.12) $$\lambda = \left(\frac{meters}{\text{sec}}\right) \times \left(\frac{\text{sec}}{cycle}\right)$$

The "seconds" cancel out:

(1.13) $$\lambda = \left(\frac{meters}{\cancel{\text{sec}}}\right) \times \left(\frac{\cancel{\text{sec}}}{cycle}\right)$$

and we are left with:

(1.14) $$\lambda = \left(\frac{meters}{cycle}\right)$$

which is the definition of wavelength i.e., wavelength is measured in units of length per cycle.

Actual values for velocity and frequency can be inserted along with units into eqns. (1.11) or (1.12) to calculate wavelength. Let's say we want to know the (free-space) wavelength of a 400 MHz. signal. Substituting numbers and units into eqn. (1.12) we have:

(1.15) $$\lambda = \left(\frac{300 \times 10^6 \, meters}{\text{second}} \right) \times \left(\frac{\text{second}}{400 \times 10^6 \, cycles} \right)$$

Where Velocity is the free-space speed of light (approximately 300,000,000 meters/second, about 186,000 miles per second), and frequency is 400,000,000 Hz. Canceling units (and powers of ten):

(1.16) $$\lambda = \left(\frac{300 \times \cancel{10^6} \, meters}{\cancel{\text{second}}} \right) \times \left(\frac{\cancel{\text{second}}}{400 \times \cancel{10^6} \, cycles} \right)$$

(1.17) $$\lambda = \left(\frac{300 meters}{400 cycles} \right)$$

(1.18) $$\lambda = 0.75 meters / cycle$$

Voila! The free-space wavelength of a 400MHz signal is 0.75 meters.

Conversion Factors

The last two examples will illustrate Conversion Factors:

Example #3

We all know, for example, that there are 12 inches in one foot. A conversion factor for changing inches to feet, or feet to inches, would be 12inches/foot or it's reciprocal, one foot "per" every 12inches written:

(1.19) $$\frac{12 inches}{foot}$$

or,

(1.20) $$\frac{1 foot}{12 inches}$$

If we want to convert, say, 66 inches into feet:

(1.21) $$66 inches = n\, feet$$

Use conversion factor (1.20) to place "inches" in the denominator for cancellation:

(1.22) $$\left(66\cancel{inches}\right)\times\left(\frac{1\,foot}{12\cancel{inches}}\right)=n\,feet$$

(1.23) $$\left(\frac{66\,feet}{12}\right)=n\,feet$$

(1.24) $$5.5\,feet=n\,feet$$

To convert feet into inches, use reciprocal factor (eqn. 1.19) to place "feet" in the denominator for cancellation:

(1.25) $$5.5\,feet=m\,inches$$

(1.26) $$\left(\frac{5.5\cancel{feet}}{1}\right)\times\left(\frac{12 inches}{\cancel{foot}}\right)=m\,inches$$

(1.27) $$\left(\frac{5.5}{1}\right)\times\left(\frac{12 inches}{1}\right)=m\,inches$$

(1.28) $$66 inches=m\,inches$$

EXAMPLE #4:

The last example shows how multiple conversion factors (Rule D) can be strung together to get the desired result:

Let's say we want to convert 40 inches to centimeters.

(1.29) $$40 inches=y\,cm$$

The conversion factor, and it's reciprocal, for converting between inches and meters are:

(1.30) $$\frac{39.37 inches}{meter}$$

(1.31) $$\frac{1 meter}{39.97 inches}$$

Since we want to cancel out inches, use conversion factor (1.31):

(1.32) $$\left(40\cancel{inches}\right)\times\left(\frac{1meter}{39.37\cancel{inches}}\right)=y\,cm$$

Note that “inches” cancels out, but we are left with “meters” on the left side of the equation; but what we really want is centimeters. So, since there are 100cm in a meter, we can add in a meter/cm conversion factor into (1.32) to cancel out meters and leave centimeters: (By the way, inserting a “conversion factor,” where equivalent units are in both the numerator and denominator, is essentially multiplying by “one,” so you can add them in as desired without changing the equation at all.)

(1.33) $$\left(\frac{40\cancel{inches}}{1}\right)\times\left(\frac{1\cancel{meter}}{39.37\cancel{inches}}\right)\times\left(\frac{100cm}{\cancel{meter}}\right)=y\,cm$$

(1.34) $$\left(\frac{40}{1}\right)\times\left(\frac{1}{39.97}\right)\times\left(\frac{100cm}{1}\right)=y\,cm$$

(1.35) $$100.08\,cm=y\,cm$$

So, 40 inches=100.08cm

A couple of other conversion factors are listed below. You can use your own (and/or their reciprocals) to do computations and conversions as you wish.

$$\left(\frac{1000mm}{meter}\right)\quad\left(\frac{mile}{5280\,feet}\right)\quad\left(\frac{1000mA}{Ampere}\right)\quad\left(\frac{1yard}{3\,feet}\right)\quad\left(\frac{2.54cm}{inch}\right)$$

By the way, the last one: (2.54cm/inch) is identical to (39.97inches/meter). Just crunch the numbers!

A Note on “Unit Analysis” and “Dimensional Analysis”:

After writing this blurb (go figure…) I was looking up some related material on the Internet, and found out that the “Units Method” is also known as Unit Analysis, Dimensional Analysis, Factor-Label Method or Unit-Factor Method. So, it “ain’t new with me.” I’ve used it for years, sort-of stumbling on it trying to keep my own calculations straight. But it’s also taught in schools (I musta been absent that day) so there’s more info out there should you need it. Hope this procedure helps.

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